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On Chromatic Effects in Observations of the Sun near the Horizon

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Abstract—The observability of the "green flash" in observations of the Sun near the horizon was calculated. The calculations were conducted with consideration for chromatic refraction, atmosphere transmission, spectral sensitivity of the eye, and image blurring due to the instability and inhomogeneity of the atmosphere.

Keywords: the Sun, astronomical observations, horizon, chromatic refraction, green ray, green flash

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INTRODUCTION

Chromatic refraction is a well-known phenomenon attributed to the wavelength dependence of the refractive index of a medium. In the context of astronomical observations from the surface of the Earth, the atmosphere acts as such a medium. The difference in the refractive indices of the terrestrial atmosphere for blue (450 nm) and red (700 nm) wavelengths is about 10^{-6} and is usually not taken into account in the observations and astrometric refraction is calculated using the refractive index for a certain middle wavelength. At the same time, the chromatic refraction effect may affect the visible image under certain special conditions (e.g., in the observation of objects at large zenith angles). This is explained by the fact that the difference in the absolute refraction angles for rays with frequencies lying at the opposite sides of the visible spectrum becomes significant at large atmospheric masses and establishes conditions for the observation of a natural phenomenon known as the green flash or green ray. The observations of bright green flashes right after sunset have been reported since the days of old. Several theories on the nature of this phenomenon were proposed, but it was later found that chromatic refraction is the deciding factor in this case. The image obtained in observations near the horizon is blurred strongly due to the instability and inhomogeneity of the atmosphere. Therefore, the green flash is a very rare phenomenon that is sensitive to atmospheric conditions, and it might be of interest to determine (by means of model analysis) the atmospheric conditions that are most favorable for this phenomenon.

A great number of factors should be taken into account when modeling the green flash. The model image of the Sun will depend not only on the choice of the atmosphere model allowing for refraction, but also on the solar limb darkening, the spectral sensitivity of the eye or photodetector, and the atmospheric turbulence. All the abovementioned factors were taken into account in our calculations in this paper. The atmospheric model and the method for calculating the atmospheric mass at large zenith angles were chosen first. Other than these, the solar limb darkening effect was taken into account because the green flash is observed at the very edge of the solar disk. The image of the Sun at different wavelengths was constructed within the framework of this model. Every detector, be it the human eye or a CCD camera, is characterized by its own spectral sensitivity. The atmosphere transmission function also varies greatly with wavelength. Since these specifics affect the relative variation of the perceived intensity of different wavelengths and, consequently, the perceived color, they were taken into account in the modeling. It was already noted that the image obtained in observations near the horizon is distorted significantly due to the instability and inhomogeneity of the atmosphere; therefore, a parameter that describes the intensity of atmospheric disturbances was also introduced. A series of model images of the Sun corresponding to different values of this parameter was obtained and the conditions for seeing the green flash were analyzed.

METHOD

We attempt to evaluate the chromatic effects occurring near the horizon. A model image of the Sun near the horizon was constructed within the framework of our study, and the effects occurring at the upper edge of the solar disk were analyzed. The algorithm for constructing the image comprises the following main steps. An exoatmospheric image of the Sun is constructed with consideration for the solar limb darkening. The images of the Sun at different wavelengths are shifted relative to each other as the radiation traverses the atmosphere towards the observer on the surface of the Earth. The blurring of the images of the Sun due to

Physical parameters used in the modeling

r_e	Radius of the Earth	6378390 m
g	Gravitational constant	9.80655 ms^{-2}
R	Specific gas constant for air	287.053 m^2 s^{-2} °C^{-1}
n	Polytropic index of the troposphere	5
h_w	Altitude of the boundary between the troposphere and the stratosphere	11019 m

atmospheric turbulence is taken into account. The variation of intensity of the images of the Sun at different wavelengths attributed to the differing atmospheric absorption coefficients and the spectral sensitivity of the image detector (the human eye) were also taken into account. The observed end color was calculated both in the XYZ and RGB color spaces [1].

The calculation of absolute refraction in observations at large zenith angles poses a rather difficult problem. The authors of [2] proposed a method for calculating the absolute refraction for any zenith angles. This idea consists of numerically solving the differential equation that relates refraction to the atmosphere density at different altitudes. In addition to this, the authors of [2] presented a two-layer atmospheric model that was used to model the image of the Sun near the horizon in our study. The atmosphere in the proposed model is assumed to be polytropic and consisting of two layers (troposphere and stratosphere) with a boundary at an altitude of h_w. The pressure and temperature distribution is postulated to be continuous at this boundary between the troposphere and the stratosphere.

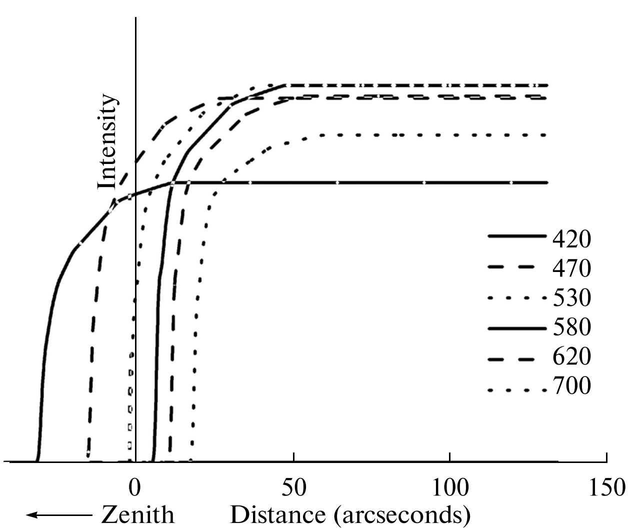

Fig. 1. Dependence of the radiation intensity on the distance from the edge of the solar disk at different wavelengths (420–700 nm) with consideration for the chromatic refraction effect.

Expressions (1) and (2) describe the height dependence of the troposphere density and the stratosphere density, respectively:

$$\rho(r) = \rho_w \left[1 + \beta_w \left(\frac{1}{r} - \frac{1}{r_w}\right)\right]^n, \quad (1)$$

$$\rho(r) = \rho_w \exp\left[\gamma_w \left(\frac{1}{r} - \frac{1}{r_w}\right)\right], \quad (2)$$

where

$$\rho_w = \frac{p_w}{760}\frac{273.15}{T_w},$$

$$\beta_w = \frac{gr_e}{[RT_w(1+n)]}, \quad \gamma_w = \frac{gr_e}{RT_w},$$

$$r = \frac{r_e + h}{r_e}, \quad r_w = \frac{r_e + h_w}{r_e}.$$

The physical constants used in the expressions above are listed in the table.

In order to evaluate the chromatic effects, we took into account the dependence of the refractive index (μ) on the wavelength (λ) using the following formula from [3] (λ is expressed in micrometers):

$$\mu = 1 + \alpha(\lambda)\rho,$$

$$\alpha(\lambda) \times 10^8 = \left\{6432.8 + \frac{2949810}{146 - (\lambda^{-2})} + \frac{25540}{41 - (\lambda^{-2})}\right\}.$$

Based on the assumptions mentioned above, we calculated the absolute refraction for different waves on the premise that the upper edge of the solar disk has a zenith angle of 90°. The data on the radiation intensity near the edge of the solar disk were obtained based on [4, 5]. The analysis undertaken allowed us to plot the dependences of intensity on the distance from the upper edge of the solar disk for waves of the visible spectrum with different wavelengths (Fig. 1). The upper edge of the solar disk with account for refraction at the green light wavelength of 530 nm was used as a reference point for the measurement of all distances in the graphs. The calculations show that a wavelength reduction results in an increase in shift due to refraction in the atmosphere. The difference between the absolute refraction angles for violet (380 nm) and red (740 nm) wavelengths equals 70″. Therefore, the chromatic effects are the most pronounced within this range. A transition of the main part of the solar disk from blue at its upper edge to white could be observed

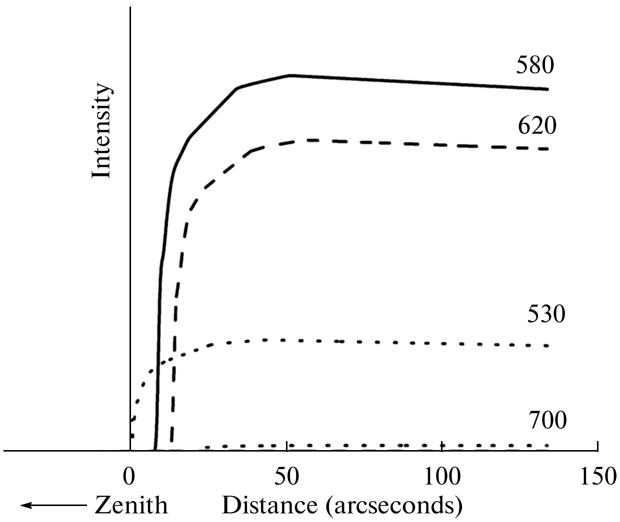

Fig. 2. Dependence of the radiation intensity on the distance from the edge of the solar disk at different wavelengths with consideration for the effect of the atmosphere transmission and the spectral sensitivity of the eye.

in an absolutely transparent atmosphere. The real color of the setting Sun is close to red, which is related to atmosphere transmission and the spectral sensitivity of the photodetector.

The atmosphere transmission in the visible wavelength range may be expressed using the Beer–Lambert–Bouguer law [6]:

$$T(\lambda, \delta) = \exp\left[-m(\delta)\tau_t(\lambda)\right],$$

where $m(\delta)$ is the air mass at the angle of elevation of the observed object δ and $\tau_t(\lambda)$ is the total optical depth of the atmosphere in the zenith direction. The total optical depth is a combination of three major components: the Rayleigh optical depth attributed to molecular scattering in the atmosphere, the optical depth associated with reflection and absorption at disperse particles in the atmosphere, and the term related to absorption at the gaseous constituents. The algorithms for calculating each of these components are described in detail in [6]. Since one should possess detailed information on the current state of the atmosphere in order to take the effect of the last two components into account, only the Rayleigh scattering contribution was analyzed in the present paper. The classic formula $m(\delta) = \sec(90° - \delta)$, used to calculate the air mass is not accurate at large zenith angles; therefore, we used the following empirical estimate:

$$m(\delta) = \left[\sin\delta + 0.025\exp(-11.0\sin\delta)\right]^{-1}.$$

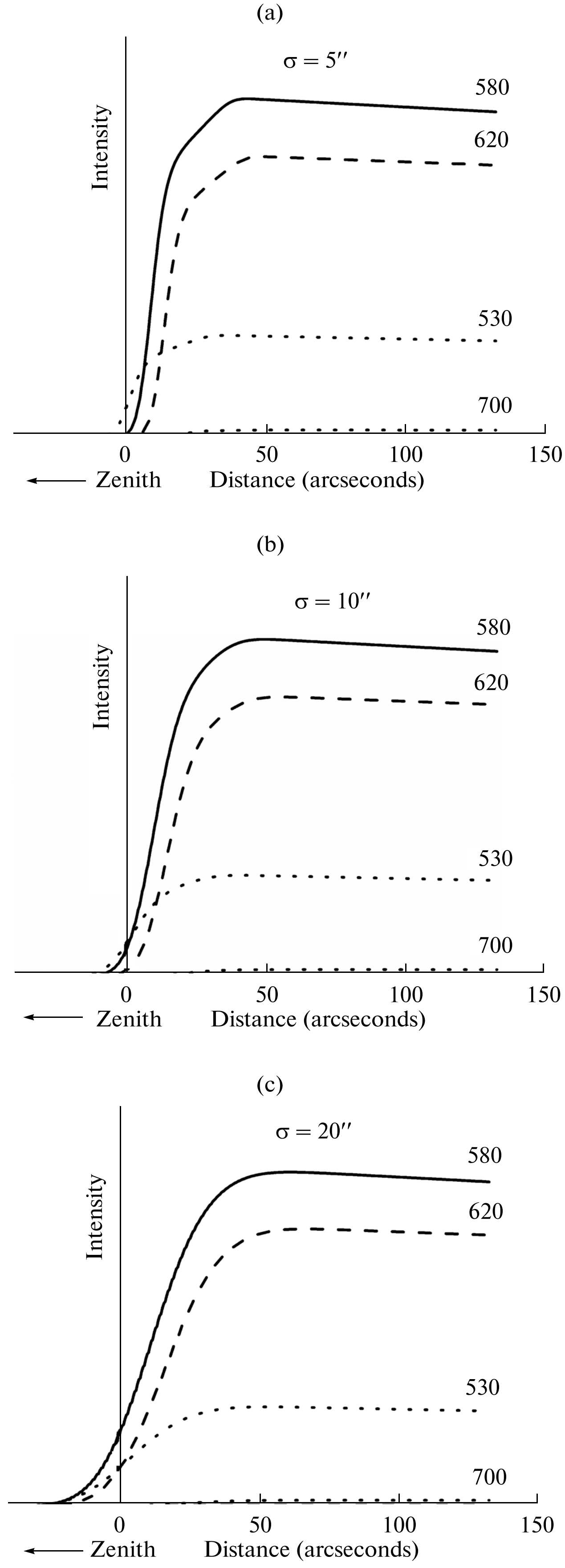

Fig. 3. Dependence of the radiation intensity on the distance from the edge of the solar disk at different wavelengths with consideration for disturbances in the atmosphere for σ = 5″ (a), σ = 10″ (b), and σ = 20″ (c).

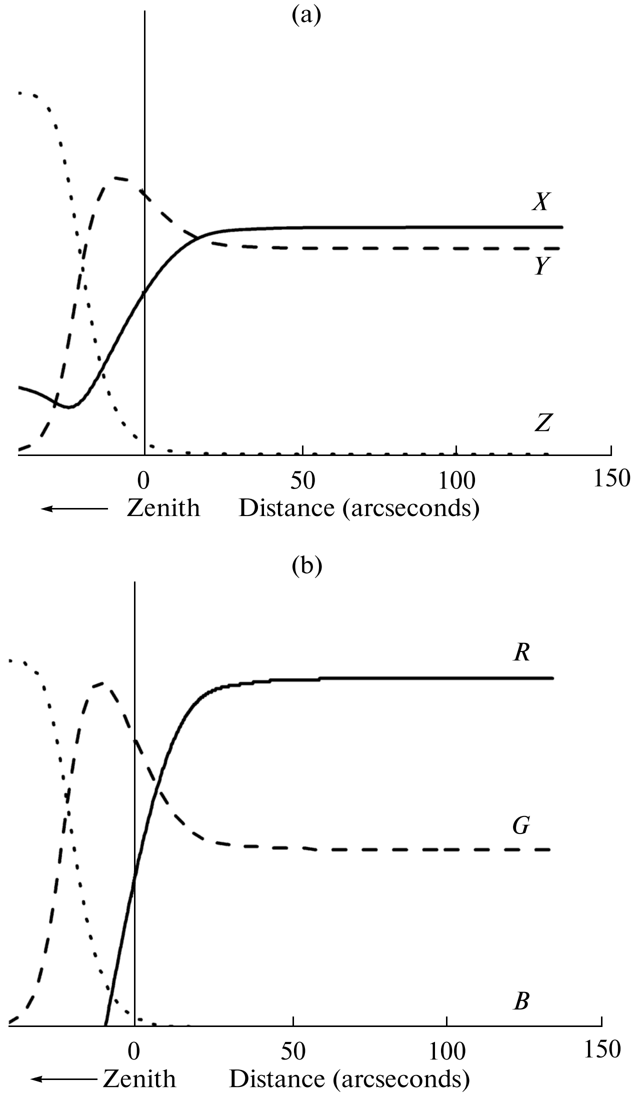

Fig. 4. Color channel values in the (a) XYZ and (b) RGB color spaces.

This formula allows one to estimate the air mass at large zenith angles in the case of a completely stationary atmosphere. The actual air mass value may vary because of the atmosphere inhomogeneity. At the same time, two factors make this very formula applicable in the case in point. First, we are most interested not in the absolute refraction of light rays, but in the difference between the angles of refraction at different wavelengths, which gives rise to the chromatic effects. The air mass estimation error may exert only a slight influence on the results of calculations carried out in the present study. Second, the blurring function introduced earlier reflects, to a certain extent, atmosphere instability; therefore, it seems permissible to neglect this factor when calculating the air mass.

The spectral sensitivity of the human eye is maximized in the 500- to 600-nm wavelength interval, which corresponds to the color green. The sensitivity of the human eye to different wavelengths is analyzed in [8]. The results of calculations carried out with consideration for the atmosphere transmission and the spectral sensitivity of the eye are shown in Fig. 2. A particular region where the intensity of waves with green wavelengths is maximized with respect to the other wavelengths is observed at the upper edge of the solar disk. This very region provides the conditions most favorable for observing the green flash. At the same time, it was already noted that the instability and inhomogeneity of the atmosphere cause image blurring and may not be neglected in actual observations conducted in the presence of a significant atmospheric mass. The instability of the atmosphere is often described with the use of the C_n^2, parameter, which defines the refractive index variations in the atmosphere. Such an approach is commonly used in atmospheric physics and requires detailed data on the state of the atmosphere. Astronomers, however, often use a different method: in order to take into account the image blurring caused by the instability and inhomogeneity of the atmosphere, they analyze the image obtained in the observation of a point source and introduce an instrument function that describes the observed blurring. According to [7], this image blurring may be described by the $I_\lambda^{(2)}(x) = \frac{1}{\sqrt{\pi\sigma}}\int_{-\infty}^{+\infty} I_\lambda^{(1)}(x-r)\exp\left(-\frac{r^2}{\sigma^2}\right)dr$, function, where $I_\lambda^{(1)}(x)$ is the function characterizing the distribution of brightness over the solar disk with distance from the chosen reference point near the edge at a wavelength of λ, $I_\lambda^{(2)}(x)$ is the function characterizing the distribution of brightness over the solar disk with distance from the chosen reference point near the edge at a wavelength of λ with consideration for the σ parameter, and σ is the parameter that characterizes the image blurring due to the atmospheric turbulence. The results obtained for different σ values are shown in Figs. 3a–3c.

It should be noted that the edge of the Sun when observed in different wavelengths is shifted even higher due to blurring. The region where the intensity of waves with green wavelengths is maximized is also narrowed down as the σ parameter is increased, and the observed color at $\sigma = 20''$ exhibits almost no dependence on the distance from the edge of the disk (i.e., conditions for observing the green flash are less favorable).

The calculations undertaken allowed us to determine the intensity values for each wavelength at each point at the upper edge of the solar disk and calculate the directly observed color at each point. In order to convert a set of intensities for different wavelengths into a color, one has to choose a certain color space. We used a 3-component CIE XYZ model [1] that takes into account the specifics of human color perception.

Unfortunately, color models do not describe the color intensity and it is impossible to tell, for example, to what extent the color green at the edge of the solar disk is brighter than the color of the main part of the disk.

Figure 4a shows the values in the three color channels of this model for the model image of the Sun obtained at $\sigma = 10''$. The XYZ colors were also converted into the classic RGB system for illustrative purposes (Fig. 4b). This figure shows even more clearly that the resulting color observed at the edge of the disk is green and thus provides an explanation for the green-flash phenomemon. Therefore, it is arguable that the conditions favorable for observing this phenomenon are established at $\sigma \leq 10''$. Let us cite for comparison the typical values of the σ parameter obtained in night-time CCD observations of stars with the MTM-500M automated telescope located at the Mountain Astronomical Station of the Pulkovo Observatory [9]: σ varies within the range of $1''$–$1.5''$ at zenith angles of up to 50°.

CONCLUSIONS

Summing up, we should mention the following results of the present study. First, the chromatic refraction effect in observations at large zenith angles may give rise to a rare phenomenon such as green flash. Second, the observability of green flash is determined by a number of other factors. One of these factors is the σ parameter characterizing the instability and inhomogeneity of the atmosphere.

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